How to observe fluctuating temperature?

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Abstract

We provide arguments that event-by-event (EBE) analysis of multiparticle production data are ideal place to search for the possible fluctuation of temperature characterizing hadronizing source in thermodynamical approach.

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In this note we would like to advertise the feasibility and importance of the observations of fluctuating temperature in high energy multiparticle production processes. Let us first list our chain of reasoning:

- ALICE will look (among other things) for the creation of Quark Gluon Plasma (QGP);
- this calls for a thermodynamical description of (at least some) relevant observables and this in turn calls for the temperature T of the system under investigation as one of the most important quantities;
- T can be deduced most directly (at least this is believed to be the case) by looking at p_T spectra because ($\mu_T = \sqrt{m^2 + p_T^2}$)

$$\frac{dN}{dp_T} \propto \exp\left(-\frac{\mu_T}{T}\right). \tag{1}$$

This is widely accepted approach, notwithstanding the fact that both details of what T really means or whether (1) is a proper form for μ_T-dependence are still subject to hot debate

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and modelling. Taking therefore (1) as our starting point we want to concentrate on question: is it possible that T is fluctuating quantity [1, 2] and if so, does it fluctuates only from event-to-event or also in a given event?

We are not going to discuss here the problem of internal consistency (or inconsistency) of the notion of fluctuations of temperature in thermodynamics referring in this matter to [3, 4, 5]. What we want to do is to brings to ones attention the fact that event-by-event analysis allows (at least *in principle*) to detect fluctuations of temperature taking place *in a given event*. This is more than indirect measure of fluctuations of T proposed some time ago in [6] or more direct fluctuations of T *from event to event* discussed in [1].

To this end let us first remind our results presented in [7] where we have shown that fluctuation of the parameter in exponential distribution leads in a natural way (under some circumstances, of course, to be mentioned in a moment) to final distribution of the power-like form (know also as Lévy distribution):

$$\left\langle \exp\left[-\left(\frac{1}{T}\right)\cdot\mu_T\right]\right\rangle \Longrightarrow \left[1+(1-q)\left(\frac{1}{T_0}\right)\cdot\mu_T\right]^{\frac{1}{1-q}}. \tag{2}$$

Here averaging $\langle\ldots\rangle$ is performed over fluctuations of parameter (here our temperature) $\frac{1}{T}$ which take place around some mean value T_0 and should follow gamma distribution (see [7] for details). The new parameter occuring here (identical to the so called entropic index or nonextensivity parameter in Tsallis statistics [8]) is tightly connected with the size of such fluctuations, namely

$$q \;=\; 1 \,+\, \omega \tag{3}$$

where

$$\omega \;=\; \frac{\left\langle\left(\frac{1}{T}\right)^2\right\rangle - \left\langle\frac{1}{T}\right\rangle^2}{\left\langle\frac{1}{T}\right\rangle^2}. \tag{4}$$

It is worth to mention that distribution of the Lévy type (2) has been observed already in inclusive processes [9]. However, inclusive processes are not able to provide unambiguous answer what is the source of such behaviour. This can be done, such is our belief, only in the careful analysis of event-by-event data, especially those for heavy ion collisions. Two scenarios are possible here and should be subjected to experimental verification:

(1) T is constant in each event but because of different initial conditions it fluctuates from event to event. In this case in each event one should find exponential dependence (1) with $T = T_{event}$ and possible departure from it will occur only after averaging over all events. It will reflect fluctuations originating in different initial conditions for each collision from which given event originates. This situation is illustrated in Fig. 1 where p_T distributions for $T = 200$ MeV (black symbols) and $T = 250$ MeV (open symbols) are presented. All other details are the same as listed below for Fig. 2. Such values of T correspond to typical uncertainties in T expected at LHC due to different initial conditions. Notice that both curves presented here are straight lines.

(2) T fluctuates in each event around some value T_0. In this case one should observe departure from the exponential behaviour already on the single event level which should be fully given (2) with $q > 1$. It reflects situation when, due to some intrinsically dynamical reasons, different parts of a given event can have different temperatures [7]. Fig. 2 shows typical event of this type obtained in simulations performed for central $Pb+Pb$ collisions taking place for beam energy equal $E_{beam} = 3$ A·TeV in which density of particles in central region (defined by rapidity window $-1.5 < y < 1.5$) is equal to $\frac{dN}{dy} = 6000$ (this is the usual value given by event generators like VENUS, SHAKER, HIJING). Black symbols represent exponential dependence obtained for $T = 200$ MeV (the same as in Fig. 1), open symbols show the power-like dependence as given by (2) with the same T and with $q = 1.05$ (notice that the corresponding curve bends slightly upward here). In this typical event we have ~ 18000 secondaries, i.e., practically the maximal possible number. Notice that points with highest p_T correspond already to single particles.

One should stress here the following important fact: our $\omega = q - 1$ has physical meaning of the total heat capacity C, because according to a basic relation of thermodynamic [4] ($\beta = \frac{1}{T}$)

$$\frac{\sigma^2(\beta)}{\langle\beta\rangle^2} = \frac{1}{C} = \omega = q - 1. \tag{5}$$

Therefore measuring in addition to the temperature T also nonextensivity q describing its fluctuation (and, because of this, the total heat capacity C) could be of great practical importance for our understanding of dynamics of heavy ion collisions [1, 2]. In particular it should not only facilitate checking the commonly made assumption that an approximate thermodynamics state is obtained in a single collision but also, by knowing the heat capacity, provide considerable information about its thermodynamics (especially on the existence and type of the possible phase transitions [1, 2]) [10].

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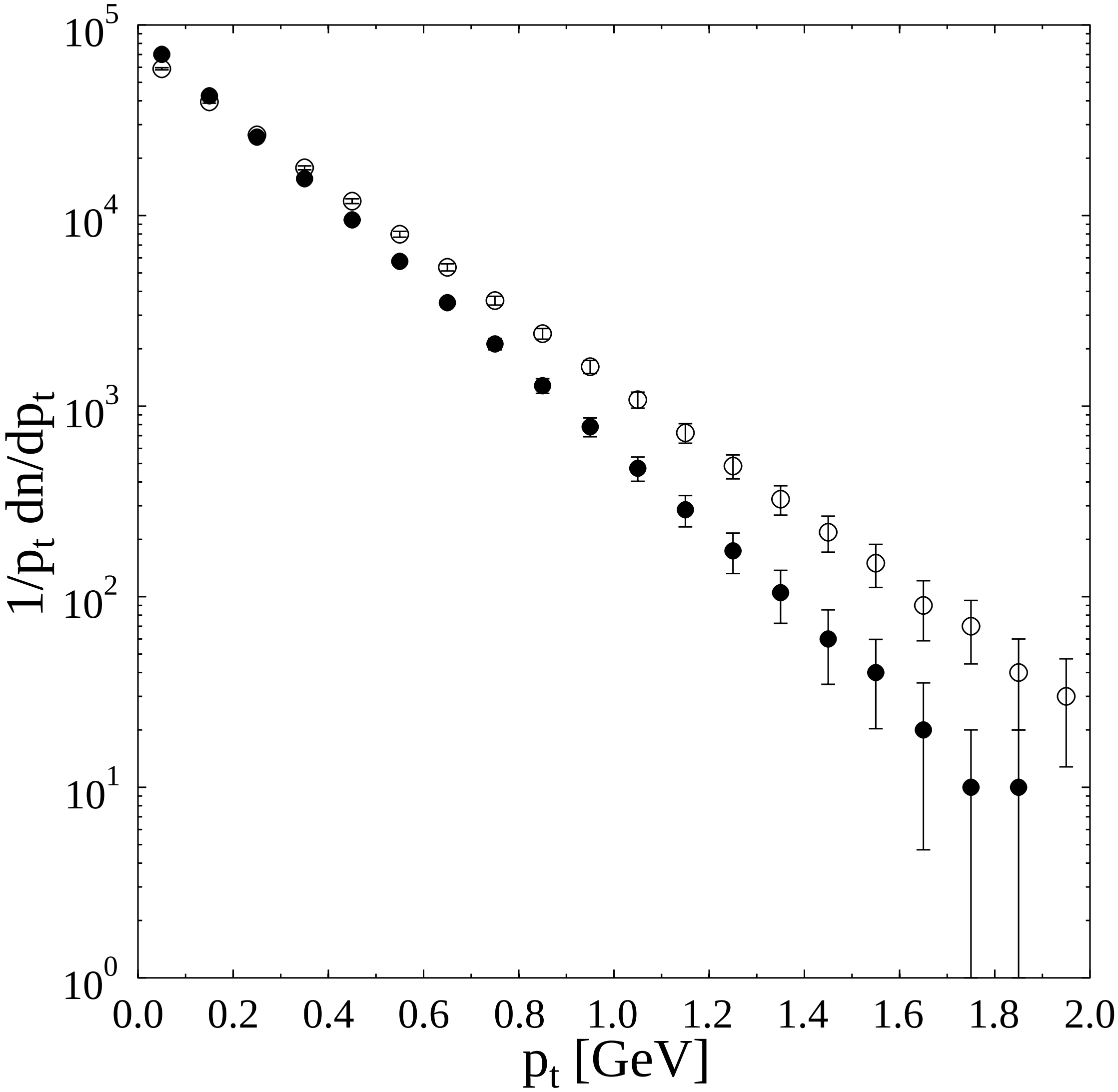
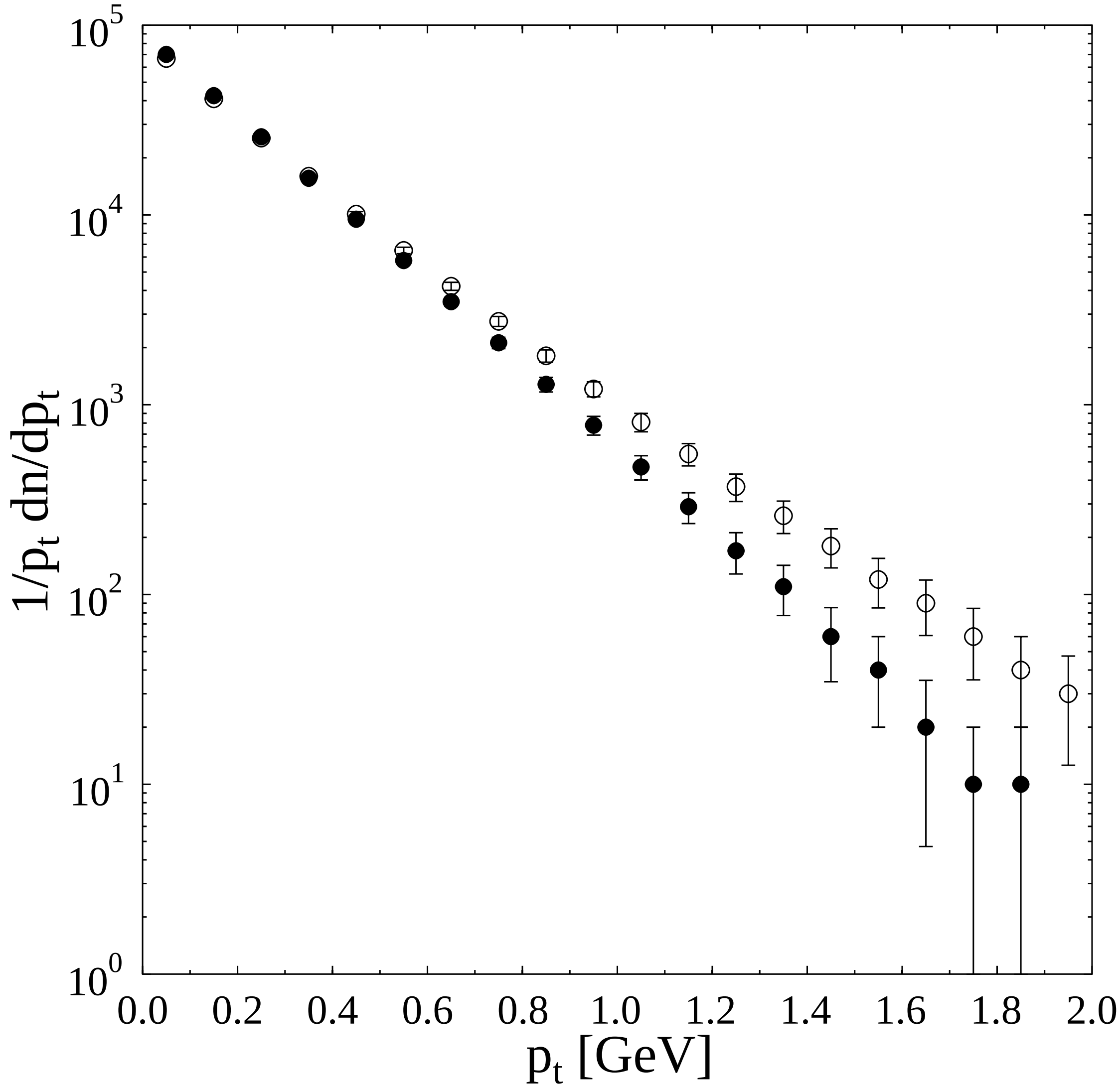
[10] It is interesting to realize that for the Planckian gas at $T = 186$ MeV, occupying volume of the order of the volume of sulfur nucleus, one gets $C = 34.4$ per degree of freedom, which leads, using (5), to $q = 1.015$ obtained for such system for the p_T dependence of produced secondaries.



10^5
10^4
10^3
10^2
10^1
10^0
$1/p_t\ dn/dp_t$
0.0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0
p_t [GeV]

Figure 1


10^5
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$1/p_t\ dn/dp_t$
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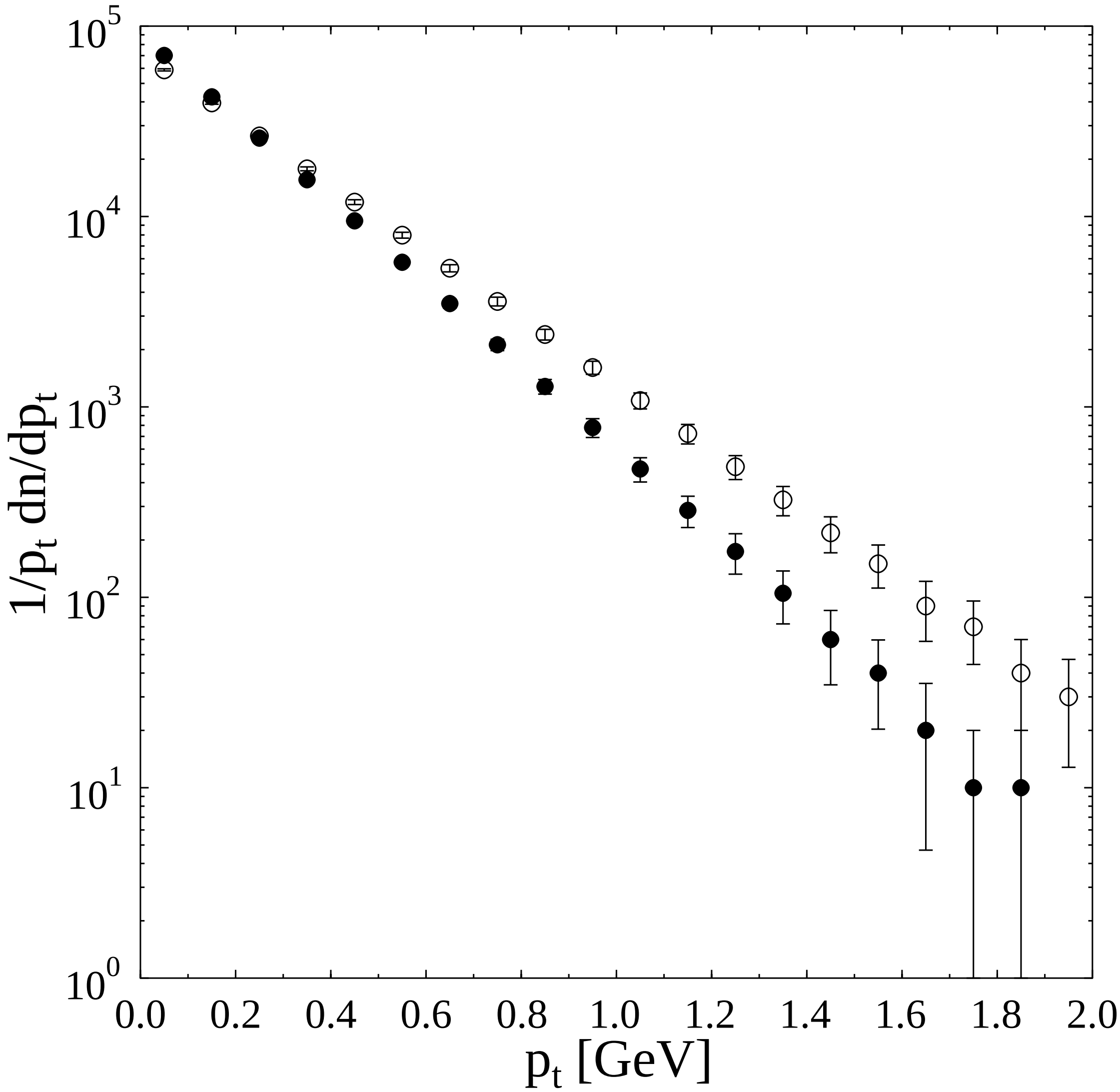
1.0
1.2
1.4
1.6
1.8
2.0
p_t [GeV]

Figure 2



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$1/p_t\ dn/dp_t$
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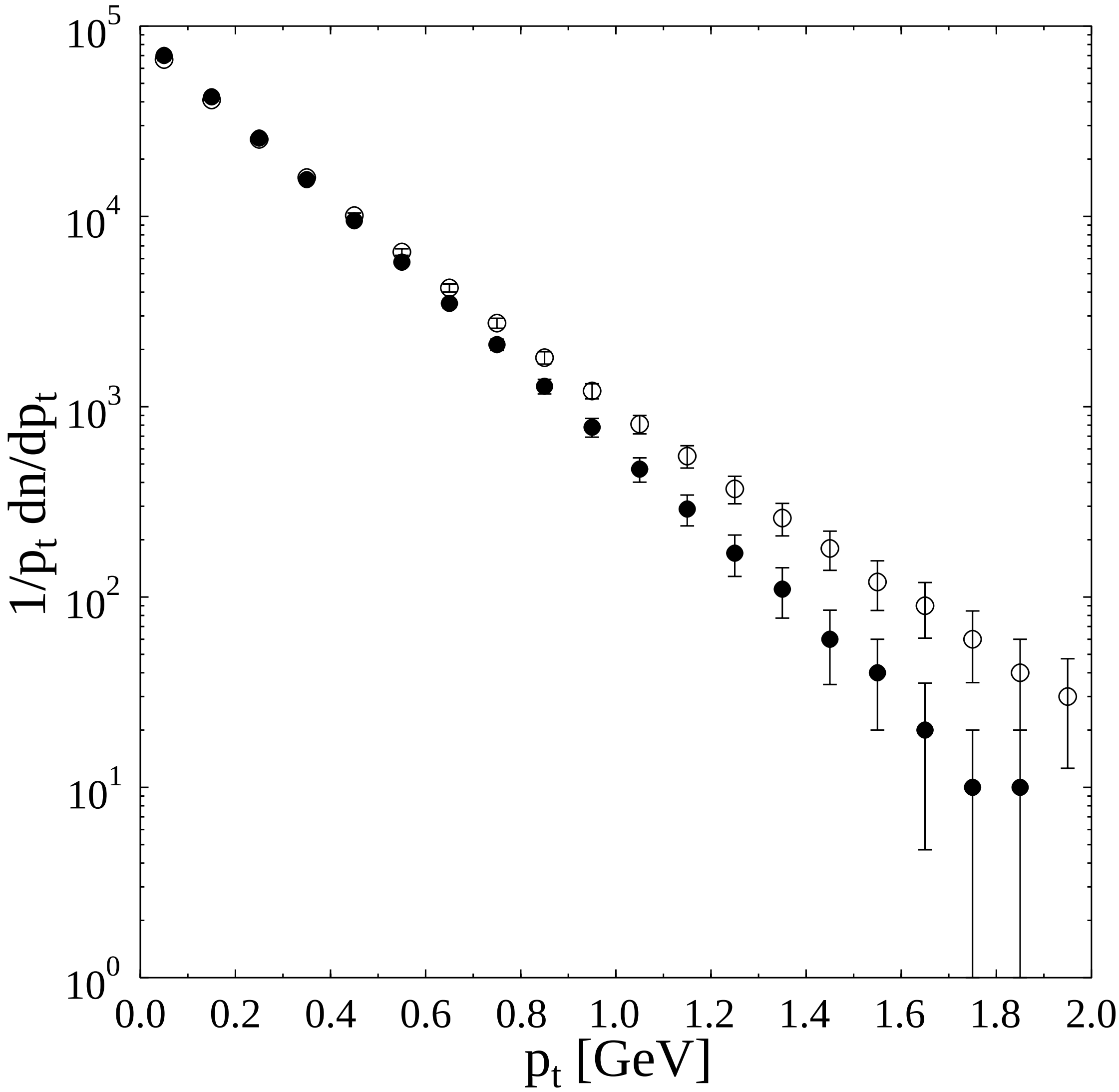
p_t [GeV]



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$1/p_t\ dn/dp_t$
p_t [GeV]